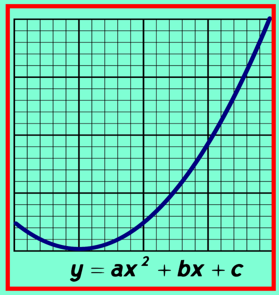
Math 125

Spring 2021

Lecture 29

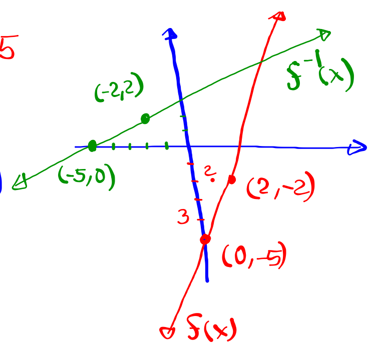


Given $f(x) = \frac{3}{2}x - 5$

Linear function

$m = \frac{3}{2}$ Y-Int $(0, -5)$

Graph $f^{-1}(x)$



Find $f^{-1}(x)$

$f(x) = \frac{3}{2}x - 5$

$y = \frac{3}{2}x - 5$

$x = \frac{3}{2}y - 5$

$2x = 3y - 10$

$2x + 10 = 3y$

$\frac{2x + 10}{3} = y$

$$\boxed{f^{-1}(x) = \frac{2x + 10}{3}}$$

Verify, we had $f(0) = -5$

$f^{-1}(-5) = \frac{2(-5) + 10}{3} = \frac{-10 + 10}{3} = \frac{0}{3} = 0$

$f(0) = -5$ ✓

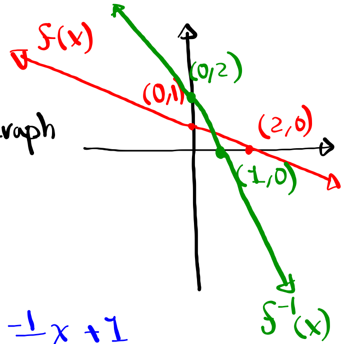
$f^{-1}(-5) = 0$ ✓

Given $f(x) = \frac{-1}{2}x + 1$

Slope $m = \frac{-1}{2}$

Y-Int $(0, 1)$

Graph



Graph $f^{-1}(x)$

Find $f^{-1}(x)$

$f(x) = \frac{-1}{2}x + 1$

$y = \frac{-1}{2}x + 1$

$x = \frac{-1}{2}y + 1$

$2x = -y + 2$

$y = -2x + 2$

$f^{-1}(x) = -2x + 2$

Verify

$f(0) = 1$ ✓

$f^{-1}(1) = -2(1) + 2$

$= -2 + 2$

$= 0$ ✓

Find $f^{-1}(x)$ for $f(x) = \sqrt{x+4}$.

$f(x) = \sqrt{x+4}$

$y = \sqrt{x+4}$

$x = \sqrt{y+4}$

$x^2 = (\sqrt{y+4})^2$

$x^2 = y + 4$

$y = x^2 - 4$

$\boxed{f^{-1}(x) = x^2 - 4}$

verify $f^{-1}(x)$ by the result of $f(5)$.

$f(5) = \sqrt{5+4} = \sqrt{9} = \boxed{3}$

$f^{-1}(3) = 3^2 - 4 = 9 - 4 = \boxed{5}$ ✓

Given $f(x) = \sqrt[3]{x-2}$

1) Find $f^{-1}(x)$

$f(x) = \sqrt[3]{x-2}$ $\quad y = \sqrt[3]{x-2}$ $\quad x = \sqrt[3]{y-2}$

$x^3 = \left(\sqrt[3]{y-2}\right)^3$ $\quad x^3 = y-2 \Rightarrow x^3+2 = y$

$f^{-1}(x) = x^3+2$

2) Verify $f^{-1}(x)$ by the result of $f(10)$.

$f(10) = \sqrt[3]{10-2} = \sqrt[3]{8} = \boxed{2}$

$f^{-1}(2) = 2^3+2 = 8+2 = \boxed{10}$ ✓

Composition Operation "f of g(x)"

$(f \circ g)(x) = f(g(x))$

f composition g of x

$f(x) = 3x-2$ $\quad g(x) = 2x+3$

Find $(f \circ g)(x) = f(g(x))$

$= 3\,g(x) - 2 = 3(2x+3) - 2$

$= 6x + 9 - 2$

$= \boxed{6x+7}$

Find $(g \circ f)(x) = g(f(x))$

$= 2\,f(x) + 3$

$= 2(3x-2) + 3 = 6x - 4 + 3$

$= \boxed{6x-1}$

$f(x) = x^2 + 2$ $\qquad$ $g(x) = \sqrt{x-2}$

Assume $x \geq 0$ for $f(x)$ and $x \geq 2$ for $g(x)$.

Find $(f \circ g)(x) = f(g(x))$

$= [g(x)]^2 + 2$

$= [\sqrt{x-2}]^2 + 2 = x \cancel{-2} \cancel{+2} = \boxed{x}$

Find $(g \circ f)(x) = g(f(x))$

$= \sqrt{f(x) - 2} = \sqrt{x^2+2-2} = \sqrt{x^2} = \boxed{x}$

$f(x) = \frac{1}{3}x - 4$ $\qquad$ $g(x) = 3x + 12$

1) Find $(f \circ g)(x)$

$(f \circ g)(x) = f(g(x)) = \frac{1}{3}\, g(x) - 4$

$= \frac{1}{3}(3x + 12) - 4$

$= \frac{1}{3} \cdot 3x + \frac{1}{3} \cdot 12 - 4$

$= x \cancel{+4} \cancel{-4} = \boxed{x}$

2) Find $(g \circ f)(x) = g(f(x))$

$= 3\, f(x) + 12$

$= 3\left(\frac{1}{3}x - 4\right) + 12$

$= 3 \cdot \frac{1}{3}x - 3 \cdot 4 + 12$

$= x \cancel{-12} \cancel{+12} = \boxed{x}$

If $(f\circ g)(x)=x$ and $(g\circ f)(x)=x$, then $f(x)$ and $g(x)$ are inverse of each other.

Ex: $f(x)=2x-3$ $\quad g(x)=\frac{x+3}{2}$

Find $(f\circ g)(x) = f(g(x)) = 2g(x)-3 = 2\left(\frac{x+3}{2}\right)-3$

$= x+3-3 = \boxed{x}$

Find $(g\circ f)(x) = g(f(x)) = \frac{f(x)+3}{2} = \frac{2x-3+3}{2}$

$= \frac{2x}{2} = \boxed{x}$

What do you conclude? $(f\circ g)(x)=x$ and $(g\circ f)(x)=x$

They are inverse of each other.

$g(x)=f^{-1}(x)$

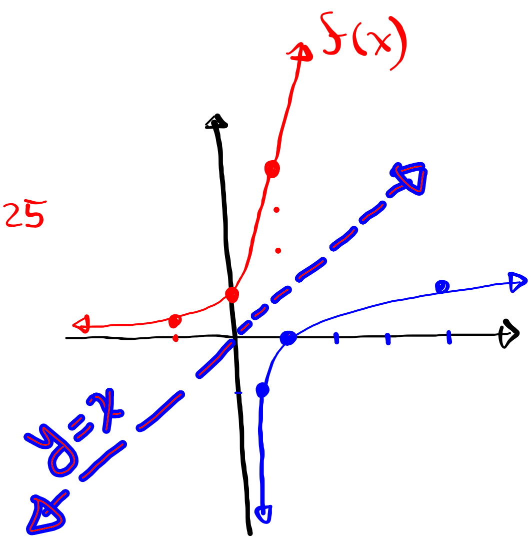
$f^{-1}(x)=g(x)$

$f(x)=4^x$

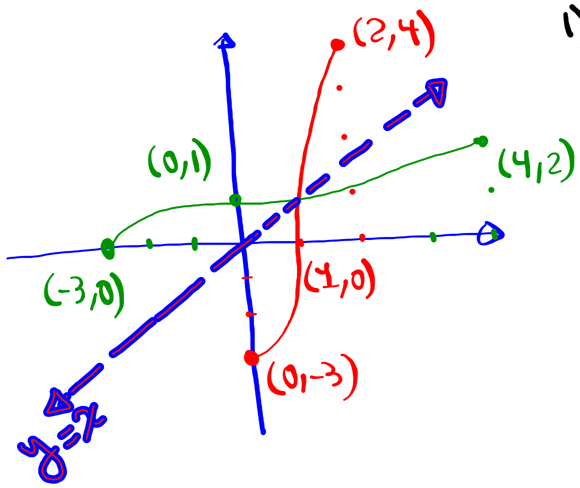
x	$f(x)$
0	1
1	4
2	16

x	$f(x)$
-1	$4^{-1}=\frac{1}{4}=.25$
-2	$4^{-2}=\frac{1}{16}$

Graph $f^{-1}(x)$



Consider the graph below



1) Domain $[0,2]$

Range $[-3,4]$

2) Graph its inverse

3) Domain & Range for the inverse.

Domain: $[-3,4]$

Range: $[0,2]$

Exponential Equations:

If $b^x = b^y$, $b>0$, $b=1$, then $x=y$.

Ex: Solve $2^{x-1}=16$

$2^{x-1} = 2^4 \Rightarrow x-1=4$

$\boxed{x=5}$ $\{5\}$

Ex: Solve $3^{2x+5}=81$

$\boxed{3}^{2x+5} = \boxed{3}^4$

$2x+5=4$

$2x=4-5$

$2x=-1$

$\boxed{x=-\frac{1}{2}}$

$\left\{-\frac{1}{2}\right\}$

Ex: Solve $4^{x^2-4x} = 64$

Hint: $64 = 4^3$

$4^{x^2-4x} = 4^3$

$x^2-4x = 3$

$x^2-4x+4 = 3+4$

$\frac{1}{2}\cdot(-4) = -2$

$(x-2)^2 = 7$

$x-2 = \pm\sqrt{7}$

$\boxed{x = 2 \pm \sqrt{7}}$

$\{2 \pm \sqrt{7}\}$

Solve $8^{x-3} = 4^{x+2}$

Hint:

$8 = 2^3$

$4 = 2^2$

$(x^m)^n = x^{mn}$

$(2^3)^{x-3} = (2^2)^{x+2}$

$2^{3(x-3)} = 2^{2(x+2)}$

$3(x-3) = 2(x+2)$

$3x-9 = 2x+4$

$3x-2x = 4+9$

$\boxed{x = 13}$

$\{13\}$

Introduction to logarithm:

Logarithm is the inverse of exponential function.

In the calc., look for log & ln.

Evaluate $\log 10 = \boxed{1}$

$\ln 10 \approx \boxed{2.303}$

Evaluate $\log 2021 \approx \boxed{3.306}$

$\ln 2021 \approx \boxed{7.611}$

Solve $2^x = 75$

75 cannot be written as power of 2.

$$\text{Exponent} = \frac{\log(\text{RHS})}{\log(\text{base})}$$

$$x = \frac{\log 75}{\log 2} \qquad \boxed{x \approx 6.229} \qquad \{6.229\}$$

Solve

$3^x = 100$

100 cannot be written as power of 3.

$$\text{Exponent} = \frac{\log(\text{RHS})}{\log(\text{Base})} \qquad x = \frac{\log(100)}{\log(3)} \Rightarrow x \approx 4.192 \ \{4.192\}$$

Solve $4^{x-1} = 250$

250 cannot be written as power of 4.

$$\text{Exponent} = \frac{\log(\text{RHS})}{\log(\text{Base})}$$

$$x - 1 = \frac{\log 250}{\log 4}$$

$$x - 1 \approx 3.983$$

$$x \approx 3.983 + 1$$

$$\boxed{x \approx 4.983}$$

$\{4.983\}$

Solve $5^{x+3} = 2021$

2021 cannot be written as Power of 5.

$$\text{Exponent} = \frac{\log(\text{RHS})}{\log(\text{Base})}$$

$$x + 3 = \frac{\log(2021)}{\log(5)}$$

$$x + 3 \approx 4.729$$

$$x \approx 4.729 - 3$$

$$\boxed{x \approx 1.729}$$

$\{1.729\}$

Solve $6^{2x-7} = 50$

Hint: 50 cannot be written as power of 6.

$$\text{Exponent} = \frac{\log(\text{RHS})}{\log(\text{base})}$$

$$2x-7 = \frac{\log 50}{\log 6}$$

$$2x-7 \approx 2.183$$

$$2x \approx 2.183+7$$

$$2x \approx 9.183$$

$$x \approx \frac{9.183}{2} \qquad \boxed{x \approx 4.592}$$

$\{4.592\}$

Solve $8^{3x+1} = 12345$

Hint: 12345 cannot be written as power of 8.

$$\text{Exponent} = \frac{\log(\text{RHS})}{\log(\text{Base})}$$

$$3x+1 = \frac{\log(12345)}{\log(8)}$$

$\{1.177\}$

$$3x+1 \approx 4.531$$

$$3x \approx 3.531$$

$$x \approx \frac{3.531}{3} \qquad \boxed{x \approx 1.177}$$

Final Exam:

1) No School Wednesday

2) Thursday

Start as early as 6:30

1) Part 1 ⇒ All graphs

2) Part 2 ⇒ Everything else.

3) Review exam 1 & 2

4) Review recent materials

5) Must show work. Work must be Similar to my lectures.

6) Submission must one file for each part.

7) Procedure is the Same as exam 1 & 2.